Conditional probability cont., Independence

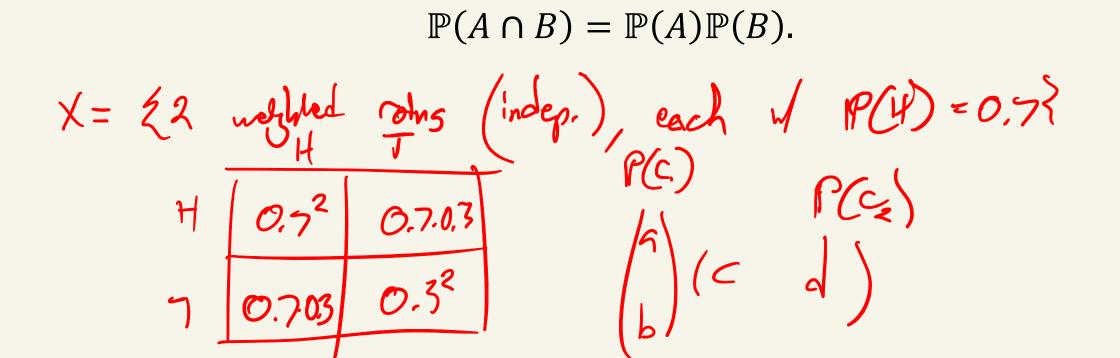
Modeling (lack of) causality

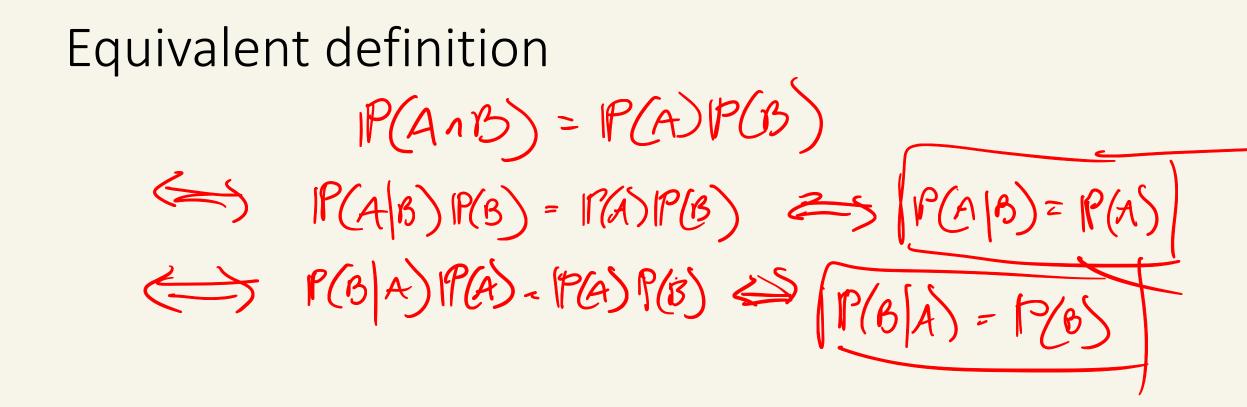
Michael Psenka

What we aim to model $X = \Xi H_{ip}$ Eg 2 cotas (2)H T, 2 $P(H, n H_{2}) = P(H)P(H_{2})$ $P(H, n T_{2}) = P(H, S)P(T_{2})$ Ha 0.25 0,25 0.25 0.22 $P(H_1 \cap H_2) \stackrel{?}{=} P(H_1)P(H_2)$ $(r) = 0.5 \cdot 0.5$ H. 0.5 Hz = 0.25

Independence

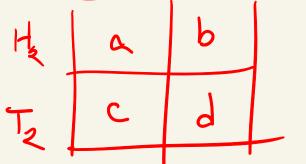
Definition. Given a random variable $X = (\Omega, \mathbb{P})$, two events $A, B \subset \Omega$ are said to be independent if the following equation holds:



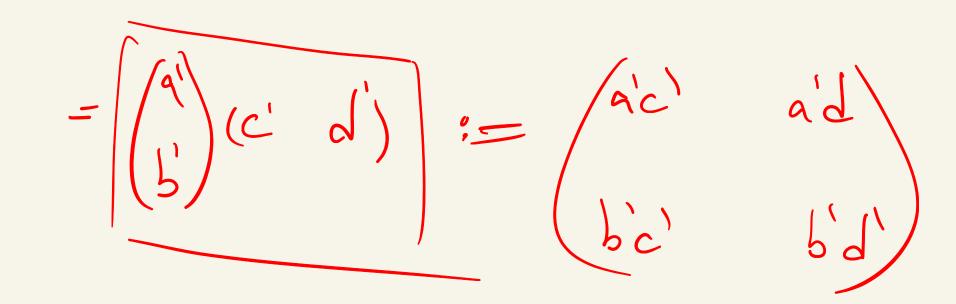


Examples E.g. die, 2= {12,3,4,5,63, P= uniform $O A = \{1,3,5\} \quad D = \{2,4,6\} \quad P(A \cap B) = P(\emptyset) = O$ $P(A) = \frac{1}{2} P(B) = \frac{1}{2} \Rightarrow P(A)P(B) = 0$ $P(A) = \frac{1}{2} P(B) = \frac{1}{2} \Rightarrow P(A)P(B) = 0$ $P(A) = \frac{1}{2} P(B) = \frac{1}{2} \Rightarrow P(A)P(B) = 0$ $P(A) = \frac{1}{2} P(B) = \frac{1}{2} \Rightarrow P(A)P(B) = 0$ $P(A) = \frac{1}{2} P(B) = \frac{1}{2} \Rightarrow \frac{$ 3 2- 812,3,42 $P(A \cap B) = P(EB) = \frac{1}{4}$ $P(A) = \frac{1}{4}, P(B) = \frac{1}{4}, \implies P(A \cap B) = P(A) P(B)$

Independence in joint distributions



 $P(H, nH_{2}) = P(H_{1})P(H_{2})$ a = (a+c) (a+b)



Mutual independence

$$\prod_{i=1}^{n} a_i \geq \alpha \cdot \alpha \cdot \cdots \cdot \alpha_n$$

$$\prod_{i=1}^{n} a_i$$

Definition. Given a random variable $X = (\Omega, \mathbb{P})$, a collection of events $A_1, \ldots, A_n \subset \Omega$ are said to be mutually independent if the following equation holds for every subset $I \subset \{1, \ldots, n\}$ such that $|I| \ge 2$:

$$\mathbb{P}(\bigcap_{i\in I} A_i) = \prod_{i\in I} \mathbb{P}(A_i).$$

 $P(A_1 \land A_2) = P(A_1) P(A_3)$ $P(A_1 \land A_3) = P(A_2) P(A_3)$ $P(A_1 \land A_3) = P(A_2) P(A_3)$

$$P(A, nA, nA_3) = P(A) P(A_3) P(A_3)$$

Why mytual independence? $\mathcal{L} = \xi H \times T \xi \times \xi H \times T \xi$ PLANBAC) = to $A = \{ \{ f^{st} \text{ coin heads} \} = \{ (H, H), (H, T) \} \}$ r B= Z2nd com bleadsz C= ¿Bolh colors some} = {CH, H), (T, T), A ! ad w/B, B ind w/C, A ind w/C bt, A,B,C mitually Indep.

General product rule

Theorem. Given a random variable $X = (\Omega, \mathbb{P})$, and a collection of events $A_1, \ldots, A_n \subset \Omega$, the following equation holds:

 $\mathbb{P}(A_1 \cap A_2 \cap \dots \cap A_n) = \mathbb{P}(A_1)\mathbb{P}(A_2 \mid A_1)\mathbb{P}(A_3 \mid A_1 \cap A_2) \cdots \mathbb{P}(A_n \mid A_1 \cap \dots \cap A_{n-1}).$

General product rule Proof Induction, Bose case, n.2, N(A, A) = N(A) P(A) IP(A) I. By definition, holds, Assume pap. holds for n-1. $\mathbb{P}(A, \Lambda_2^{n}, \Lambda_n) = \mathbb{P}((A, \Lambda_n, \Lambda_n) \wedge A_n)$ = $P(A_1, \dots, A_{n-1})P(A_n | A_1 \cap A_2 \cap \dots \cap A_{n-1})$ $= P(A_{n})P(A_{n}|A_{n})\cdots P(A_{n}|A_{n}, A_{n})$



General product rule: equivalent forms P(A, nAz nAz) = IP(Az nA, nAz) = P(A) P(A A) P(A, A, A) P(A,) P(A3/A)... $P(w_{ln}, s_w_{lkh}) = P(P_1 \cap C_2 \cap H_3) \cup (P_1 \cap C_3 \cap H_2) \cup \dots$ $P(P_1 \cap C_2 \cap H_3) + \dots$ $= P(P, nC,) * P(P_2, nC_2) + P(P_3, nC_3)$

car der hosts pick Monty Hall revisited H_1, H_2, H_3 P, P_3, P_3 C, C_8, C_3 person's cheld? $P(wm) = P(P, nC) \cup (P_2 nC_2) \cup (P_2 nC_3))$ $P(P, nC) = P(P, nC, CH) + P(P, nC, nH_2) + P(P, nC, nH_3)$ $P(H, nC_2, nP_3) = P(C_3) P(P_3|C_2) P(H, |C_2, nP_3)$ 5· 5· = 4